[This question paper contains 6 printed pages.]

Your Roll No.............

Sr. No. of Question Paper : 5800 **H**

Unique Paper Code : 237101

Name of the Paper : Probability and Statistical Methods–I

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **6** questions in all.
3. Q. No. **1** is compulsory.
4. Attempt **5** more questions selecting atleast **2** questions from each section.
5. Use of simple calculators is allowed.

1. (a) Fill in the blanks :

   (i) Bowley's coefficient of skewness $S_k = -1$, if median = ______.

(ii) The total number of class frequencies of all orders, for n attributes is ______.

(iii) Harmonic mean of first n natural numbers is ______.

(iv) For a leptokurtic distribution, γ_2 is ______.

(v) Twenty five books are placed in a shelf. Find the probability that a particular pair of books shall be always together.

(b) Find the coefficient of variation of a frequency distribution given that its mean is 120, mode is 123 and Karl Pearson's coefficient of skewness is –0.3.

(c) If the letters of the word 'RANDOM' are arranged at random, what is the probability that there is exactly one letter between A and O?

(d) If $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$. Find lower limit of $P(A \cap B)$.

(e) Define nominal and ordinal data.

(f) For a distribution, mean is 10 and variance is 16, $\gamma_1 = 1$ and $\beta_2 = 4$. Find μ_3 and μ_4.

(5,2,2,2,2,2)

SECTION – A

2. (a) Show that in a discrete series if deviations $x_i = X_i - M$, are small compared with the value of the mean M so that $(x/M)^3$ and higher powers of (x/M) are neglected,

(i) $\text{Mean}\left(\frac{1}{\sqrt{X}}\right) = \frac{1}{\sqrt{M}}\left(1 + \frac{3\sigma^2}{8M^2}\right)$ approx.

(b) Find the mean deviation about mean and standard deviation (S.D.) of A.P. a, a+d, a+2d,..., a+2nd and verify that S.D. is greater than mean deviation about mean. (6,6)

3. (a) Discuss the principle of least squares. Derive the normal equations for fitting the curve $Y = a*\exp(cX^2)$ to the given set of n points $\{(x_i, y_i), i = 1, 2 ..... n\}$.

(b) Define Yule's coefficient of association (Q) and coefficient of colligation (Y). Prove that $Q = 2Y/(1 + Y^2)$. What is the range of values for Q? (6,6)

4. (a) Establish the relationship between the moments about mean, and moments about origin. Hence obtain first four central moments in terms of moments about origin and vice-versa. What are the Sheppard's corrections to central moments?

(b) What do you mean by independence of attributes? Give a criterion of independence of attributes. If $\delta = (AB) - (AB)_0$, where $(AB)_0$ is the value of (AB) under the hypothesis that attributes A and B are independent, prove that

$$[(A)-(\alpha)][(B)-(\beta)] + 2N\delta = (AB)^2 + (\alpha\beta)^2 - (A\beta)^2 - (\alpha B)^2$$

where symbols have their usual meanings. (6,6)

5 (a) If ∂_r is the rth absolute moment about zero, then use the mean value of

$$\left\{u|x|^{(r-1)/2} + v|x|^{(r+1)/2}\right\}^2$$

to show that $\partial_r^{r+1} \le \partial_{r+1}^{r}$.

(b) The first four moments about x = 4 are 1, 4, 10, and 45. Determine the corresponding moments (i) about the mean, and (ii) about zero. (6,6)

SECTION – B

6. (a) If $A_1, A_2, \ldots, A_n$ are n independent events with $P(A_i) = 1 - \frac{1}{\alpha^i}$, i = 1, 2,...., n then find the value of $P(A_1 \cup A_2 \cup \ldots \cup A_n)$.

(b) A and B alternately cut a pack of cards and the pack is shuffled after each cut. If A starts and the game is continued until one cuts a diamond, what are the respective chances of A and B first cutting the diamond?

(c) Three newspapers A, B, and C are published in a certain city. It is estimated from the survey that of the adult population 20% read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C, 2% read all three. Find the percentage that read at least one of the papers? Also find the percentage that read both A and B but not both road C? (3,3,6)

7. (a) Define conditional probability. Further state and prove Bayes theorem.

(b) It is 8: 5 against the wife who is 40 years old living till she is 70 and 4: 3 against her husband now 50 living till he is 80. Find the probability that (i) both will be alive, (ii) only one will be alive and (iii) at least one will be alive 30 years hence. (6,6)

8. (a) Define independent events, pairwise independence, and mutual independence.

A sample contains six points $E_1, E_2, E_3, E_4, E_5, E_6$ with the probabilities $P(E_1) = 0.20$, $P(E_2) = 0.05$, $P(E_3) = 0.30$,

$P(E_4) = 0.10$, $P(E_5) = 0.10$ and $P(E_6) = 0.25$. Three events A, B and C are defined as follows :

$A = \{E_1, E_2, E_3\}$, $B = \{E_3, E_4\}$ and $C = \{E_5, E_6\}$

(i) Which pair of events A, B and C is/are mutually exclusive?

(ii) Which pair of events A, B and C is/are independent?

(iii) Are the events A, B and C mutually independent?

(b) In a class of 75 students, 15 were considered to be very intelligent, 45 as medium and the rest below average. The probability that a very intelligent student fails in a viva examination is 0.005; the medium student failing has a probability 0.05; and the corresponding probability for a below average student is 0.15. If a student is known to have passed the viva-voce examination, what is the probability that he is below average?

(6,6)